Roll No.:....

320351(14)

B. E. (Third Semester) Examination, Nov.-Dec. 2021

(New Scheme)

(Civil Engg. Branch)

MATHEMATICS-III

Time Allowed: Three hours

Maximum Marks: 80

Minimum Pass Marks: 28

Note: Attempt all questions. Part (a) from each question is compulsory and carry 2 marks.

Attempt any two parts (b), (c) & (d) with carries 7 marks each.

Unit-I

- 1. (a) Write the Dirichlet's conditions for Fourier series.
 - Expand the Fourier series for $f(x) = |\cos x|$

 $-\pi \leq x \leq \pi$.

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(c) Find the Fourier series to represent the function given by

$$f(x) = x \qquad 0 \le x \le \pi$$

$$= 2\pi - x \quad \pi \le x \le 2\pi$$

Hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

(d) The following values of y gives the displacement in inches of a certain machine part for the rotation x of flywheel. Expand y' in terms of Fourier series upto 2nd harmonic.

$$x : 0 \quad \pi/6 \quad 2\pi/6 \quad 3\pi/6 \quad 4\pi/6 \quad 5\pi/6$$

 $y : 0 \quad 9.2 \quad 14.4 \quad 17.8 \quad 17.3 \quad 11.7$

Unit-II

- 2. (a) Write the statement of convolution theorem.
 - (b) Evaluate by using Laplace transform $\int_{0}^{\infty} t^{3}e^{-t} \sin t \, dt$

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(c) Find $c^{-1} \left(\frac{s^2}{\left(s^2 + a^2\right) \left(s^2 + b^2\right)} \right)$ by using convolution theorem.

(d) Solve the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t$$

given
$$x(0) = 0$$
, $x'(0) = 1$, at $t = 0$.

Unit-III

3. (a) Form the partial differential equation by eliminating arbitary function.

$$z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$$

(b) Solve:

$$(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$$

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(c) Solve:

$$(D^2 - DD' - 2D'^2)z = (y-1)e^x$$

$$\frac{\partial u}{\partial x} = \frac{2 \partial u}{\partial t} + u$$
, where $u(x, 0) = 6e^{-3x}$.

Unit-IV

4. (a) Find the analytic function whose real part is

$$\frac{\sin 2x}{\left(\cosh 2y - \cos 2x\right)}$$

- (b) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even through C - R equation are satisfied there it.
- (c) Evaluate by using Cauchy's Integral formula $\int \frac{\log z}{(z-1)^3} dz$, where $C: |z-1| = \frac{1}{2}$.
- (d) Evaluate

$$\int_{C} \frac{\sin \pi z^{2} + \cos \pi z^{2}}{(z-1)^{2}(z-2)} dz$$

where C is the circle |z| = 3.

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- **Unit-V**
- 5. (a) Define briefly the discrete and continuous probability distribution function.

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(b) The probability density p(x) of a continuous random variable is given by

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$$p(x) = y_0 e^{-|x|}, \quad -\infty < x < \infty$$

P.T. $y_0 = 1/2$. Find the mean and variance.

(c) Fit the Poisson distribution for the following data:

(d) In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and S.D. of the distribution

$$P(z=0.5)=0.19, P(z=1.4)=0.42.$$
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